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RETROSPECT AND PROSPECT FOR MATHEMATICS IN AMERICA.

RETIRING PRESIDENTIAL ADDRESS DELIVERED BEFORE THE MATHEMATICAL ASSOCIATION OF AMERICA, SEPTEMBER 66, 1920.

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On several occasions since January, 1913, when twelve men, representing as many universities and colleges of the Middle West, took over *THE AMERICAN MATHEMATICAL MONTHLY* and dedicated it to the interests of collegiate mathematics, editorial comment in this journal has naturally taken the form of gratitude for past successes and optimism for future achievement. One of those occasions¹ was at the end of the first year of this critical undertaking when the subscription list had been doubled and it was becoming apparent that the self-imposed duties and responsibilities of that first editorial board, in their effort to render genuine service to the teachers of collegiate mathematics, were eliciting widespread approbation. Another such occasion² was at the end of three years, when the subscription list had again doubled, the subvention being no longer needed had been released, and over one thousand charter members had joined the newly organized *MATHEMATICAL ASSOCIATION OF AMERICA* with the *MONTHLY* as its official journal.

It seems highly appropriate now, near the end of the eighth year, that my remarks on the present occasion should again fall into the form of "Retrospect and Prospect," not alone directly with respect to the Association and the *MONTHLY*, but more especially with respect to recent developments in mathematical activity as a whole in America and the rôle which the Association and the *MONTHLY* may reasonably be expected to play in future developments.

First, let us recall and emphasize the fundamental things for which the Association and the *MONTHLY* stand. By the terms of its constitution the object of the Association is "to assist in promoting the interests of mathematics in America, especially in the collegiate field." This is, indeed, a broad charter, but it is subject to one obvious limitation inherent in the very circumstances under which the Association was conceived and organized, namely, that there should be no conflict in ideals between the Association and the American Mathematical Society, whose object, as stated by its constitution, is "to encourage and maintain an active interest in mathematical science." Just as the Society, by the resolution of its Council³ in April 1915, decided that it was unwise to enter into the activities then represented by *THE AMERICAN MATHEMATICAL MONTHLY*, but expressed its realization of the importance of work in that field and its

¹ "Retrospect and prospect" by H. E. Slaughter, *AMERICAN MATHEMATICAL MONTHLY*, 1913, 1-3.

² "A tentative platform of the Association" by E. R. Hedrick, *AMERICAN MATHEMATICAL MONTHLY*, 1916, 31-33.

³ *Bulletin of the American Mathematical Society*, Vol. 21, page 482.

value to mathematical science, and pledged its hearty good will and encouragement to a new organization, if such should be formed to deal specifically with this work; so the Association, on its part, concerns itself more largely with what someone has called the humanitarian aspects of mathematics and leaves to the Society the field of pure research in the technical sense of that term.

However, the Association in two or three important particulars fulfills its purpose "to assist in promoting the interests of mathematics," even in connection with research, and, indeed, thereby actually pledges its good will to the Society. Namely, (1) it has during the past three years contributed to the support of one of the American research journals at a time when such support was of vital importance, and has agreed to continue such support at least for a fourth year; (2) it has been helpful in reawakening to activity in research some who had lost interest or become discouraged through isolation or lack of sympathetic touch with others, or lack of a medium of communication whose standards were not beyond their reach; (3) it has, through the MONTHLY, consistently and persistently fostered the *beginnings of research* by helping to supply some of the lower rungs which had been lacking in the research ladder. To quote from one of the MONTHLY editorials¹: "It is by no means true that we are without interest in the higher, technical, mathematical field. On the contrary, we have an interest that is far more vital than the mere supplying of technical papers which can be read by specialists only. We believe that large numbers who would become active and effective in higher mathematical research are now lost to the cause simply by reason of the fact that there are no intermediate steps by which they can climb to these heights. We believe that the MONTHLY has a mission to perform in holding the interest of such persons by providing mathematical literature of a stimulating character that is within their range of comprehension, and by offering an appropriate medium for the publication of worthy papers which the more ambitious among them may produce." Notable examples of the far reaching effects of this sort of helpfulness can be cited.

One other fundamental position of the Association needs to be re-emphasized, namely, its relation to the *teaching* of mathematics. Many friends of the Association at the time of its organization feared that it might be in danger of degenerating into a pedagogical debating society, whose discussions might all evaporate into futility. The best guarantee at that time against such an outcome was the record already established by the MONTHLY in the preceding three years, a record which has been maintained by both the Association and the MONTHLY in the succeeding five years. The Association is, indeed, most vitally concerned with the teaching of mathematics, especially in the collegiate field, but it has very little confidence in the efficacy of mere pedagogical devices, however clever these may be, to produce and maintain high standards of teaching.

As for pedagogy in a scientific sense, its discussions belong to the realm of psychology and the research departments of education. To quote from the

¹ AMERICAN MATHEMATICAL MONTHLY, 1914, 2.

foreword to the MONTHLY¹ in 1913 and the tentative platform² of the Association in 1916: "It is not the province of the MONTHLY to enter the field of general pedagogy, nor will the MONTHLY entertain discussions that are concerned with research in general pedagogy, or that deal with new theories of pedagogy, however important these contributions may seem. . . . What we desire is to inspire, not a discussion along these lines, but rather a discussion of definite mathematical problems. . . . Just as research will be held to be within the province of the Association only if the word is given a broad interpretation, it may be said also that the discussions which this Association will foster may be termed pedagogical only if the word is used in a much broader sense than is common."

To sum up the matter now, after five years of experience, the activities of the Association bearing upon the teaching of mathematics include at least the following: (1) the presentation, discussion, and publication of papers calculated to stimulate general mathematical interest and professional esprit de corps, or of papers calculated to stimulate thoughtful consideration of possible new courses or reconsideration of the content and form of old courses; (2) the investigations and reports of committees such as the Committee on Libraries which recommended minimum lists of books for reading in the various college years, the National Committee on Mathematical Requirements, which is now engaged in a complete reconsideration of the curriculum content from the elementary school to the college, and the Committee on Mathematical Dictionary which desires to see in the hands of every teacher and student of mathematics a source of information giving in clear, compact, and scientific form an explanatory definition of every mathematical term which is likely to be found in their reading or study.

It should be added in this connection that the foregoing remarks concerning technical pedagogical matters refer to the formal meetings of the Association and formal papers in the MONTHLY. The numerous sections of the Association provide sufficiently small units to give full opportunity for informal discussions of any character and to any extent that may be deemed desirable by the members concerned; and the department of Questions and Discussions in the MONTHLY provides an open forum for free discussion in brief form of any questions deemed by the editors to be of interest and value.

Turning now to the broader retrospect of American Mathematical activities, we find the field well classified by Professor Fiske in his presidential address in 1904, namely, including the colonial period, up to the founding of Johns Hopkins University in 1876, the period from 1876 to the nationalizing of the New York Mathematical Society in 1891, and the period from 1891 to 1904. Our retrospect need not extend back of this latter date, except that it will be convenient to consider here the entire twenty year period from 1900 to 1920. The mathematical development during this period may be gauged pretty accurately on the scientific side by the activities of the American Mathematical Society and by the rapid expansion of the American colleges and universities; and on the teaching side by the activities of the secondary associations and more recently of the Mathe-

¹ Vol. 20, 1913, 4.

² Vol. 21, 1916, 33.

mathematical Association of America, and by the rapid expansion of the American high schools, both within their own original boundaries and also downward to include the junior high schools and upward to include the junior colleges.

During this twenty-year period the Society has held six of its eight colloquia¹ at which series of from four to six lectures each were delivered by Professors Oskar Bolza, E. W. Brown, E. B. Van Vleck, H. S. White, F. S. Woods, E. H. Moore, Max Mason, E. J. Wilczynski, G. A. Bliss, Edward Kasner, L. E. Dickson, W. F. Osgood, Oswald Veblen, and G. C. Evans. Several of these colloquia lectures were subsequently published in book form. The attendance upon these lectures has averaged 48, the highest being 69 at Cambridge in 1916.²

The Society has held during this period twenty summer meetings, with an average attendance of 48 and a maximum of 80 at Ann Arbor in 1919, at which 606 scientific papers were read; twenty February meetings and twenty October meetings in New York, with an average attendance of 34 at which a total of 471 papers were read; eighteen December meetings in the east, chiefly in New York, with an average attendance of 68 and a maximum of 131; and twenty December meetings in the middle West, chiefly in Chicago, with an average attendance of 43 and a maximum of 84, at which 514 and 407 papers respectively were read; nineteen April meetings in New York with an average attendance of 49 and a maximum of 82; twenty April meetings in Chicago with an average attendance of 41 and a maximum of 69, at which 422 and 442 papers respectively were read; thirty-five meetings of the San Francisco Section with an average attendance of 14 at which 299 papers were read; and twelve meetings of the Southwestern Section with an average attendance of 18, at which 212 papers were read. The total number of meetings held by the Society and its sections during this period is thus 184 and the number of scientific papers read is 3373. Of these papers 2155 had been published when the annual list for 1919 was made up.

While this conspectus of meetings of the Society manifestly gives only a quantitative measure of some of the scientific activity of American mathematicians, it nevertheless provides a fairly definite indication of progress in research work. The lists of papers read before the Society and subsequently published show an average annual output for the twenty years of 108 papers. These papers and numerous others read before other scientific bodies such as local and state academies of science, the National Academy of Science, national and international congresses, together with monographs and treatises published by educational foundations, by universities, or by commercial publishers, together constitute a scientific output in mathematics which not only makes a favorable showing for the past but also augurs well for the future.

Just as the attendance upon meetings and the output of scientific papers provide a quantitative measure of the contribution to mathematical progress made through the American Mathematical Society, so the number of doctorates

¹ Not including the Evanston Colloquium in 1893 at which Professor Felix Klein delivered a series of lectures.

² The ninth colloquium was since held at Chicago with an attendance of 88.

in mathematics, with their rate of increase and distribution, furnishes a certain quantitative measure of the contribution made by the universities. From 1898 to 1908 the average yearly number of doctorates in mathematics was about 12, while from 1908 to 1918 the average more than doubled. In 1900 only five universities granted the doctor's degree in mathematics, and to a total of eleven candidates. In 1916, thirteen universities granted the degree to a total of 35 candidates, the largest number in any year up to that time. While it is true that more than three-fourths of all the doctorates in mathematics from 1900 to 1920 have been given by eight universities, namely, Chicago 93, Johns Hopkins 39, Yale 38, Harvard 37, Cornell 31, Columbia 30, Pennsylvania 21, and Clark 16, a total of 305, nevertheless, twenty-nine different universities have contributed to the grand total of 406 mathematical doctorates during this period. It was approximately in 1910 that several other institutions began seriously to make their contributions to such doctorates. These are notably, Illinois with a total to date of 17, California with 15, Princeton with 15, and Michigan with 10. Other universities with more than one doctorate in this period are: Wisconsin 7, Syracuse 6, Indiana 5, Bryn Mawr 5, Virginia 4, Catholic University 3, Boston, Missouri, and Kansas, each 2.

We thus have had eight universities steadily and consistently throughout the twenty years contributing highly trained men and women to the mathematical faculties of our institutions, and four other universities doing likewise during the past ten years, while still others are showing signs of similar activity. The significance of these figures is not so much in the actual output, for this is small indeed compared to the needs, as in the portent for the future. Not one of the twelve universities just mentioned has reached anything like its limit of capacity for turning out trained men, while at least another dozen may be added to the productive list in the next decade, stimulated, as they are bound to be, by the cumulative influence of the steadily increasing body of trained mathematicians. What has been said with respect to mathematics is equally true of practically all other sciences, and in some cases with greater emphasis, so that the combined influence of the rapidly growing body of trained scientific men is certain to elevate at least a score of American universities to high rank in the production of research scholarship in the next twenty years.

Having deliberately laid primary emphasis upon the development of research scholarship in this country during the past twenty years, let us now briefly review the activities of this period with respect to the improvement of mathematical teaching. It is not too much to say that the presidential address of E. H. Moore in December, 1902, started a train of thought and action whose far-reaching influence has extended through all these years to the present time. Its first immediate effect was the expansion of the Central Association of Physics Teachers, then just organized in Chicago, into the Central Association of Science and Mathematics Teachers, an organization which has always been and is now at the very front in the promulgation of progressive ideas on the teaching of secondary mathematics. Similar organizations at once followed in 1903 in New

England and in the Middle States and Maryland, and a rapid succession of state and local organizations ensued, especially in the Middle West, and reaching to the Rocky Mountains and the Pacific Coast. There seemed to be a spontaneous awakening and a universal desire to reconsider the very fundamentals of procedure in the teaching of mathematics. There followed at varying periods committee investigations¹ and reports on geometry, on algebra, on combined mathematics, on mathematics for students of engineering, and on all phases of facts concerning the teaching of mathematics in this country gathered and summarized by the fourteen American committees under the International Commission on the Teaching of Mathematics.

All of these activities are in a sense now centered in the National Committee on Mathematical Requirements which was organized by the Mathematical Association of America as one of its earliest official actions, and which later became still more truly representative of national interests when it was enlarged to include members from the three great secondary associations of mathematics teachers and from the southwestern and Pacific coast sections. This Committee is unique in several respects. It is adequately financed by the General Education Board which, after careful investigation, considered the work of the Committee of sufficient national importance to warrant a liberal appropriation of funds for its use last year and a largely increased appropriation for a second year. The committee is able, therefore, to hold periodic meetings with all members present; it can command the services of a chairman and vice-chairman who give their whole time and attention to the weighty questions with which it is concerned; it can keep in touch with all the organizations of mathematics teachers in the country, not merely by correspondence, but especially by personal contact in their meetings, thus gathering at first hand the results of cooperative activity from all sources. The scope of this committee's activity is unlimited. While it is at present engaged in consideration of the secondary field, it is bound eventually to reach down into the junior high school and elementary school curricula, and to reach upward into the junior college domain. Finally, this committee enjoys the confidence of the Bureau of Education at Washington, so that its reports are published as public documents.

Never before in connection with mathematics in America has there been a committee with such an organization and such an opportunity for effective action. Possibly never before has there been greater need of just such an organized body of friends of mathematics as this committee represents. The whole high-school curriculum has been called in question by would-be reformers who would seek to justify the retention of any subject in the curriculum only in so far as its direct usefulness in the practical affairs of life can be established. Mathematics has come in for its full share of criticism and by many has been weighed in the balance to their own satisfaction and found wanting. It is maintained by them that on the score of practical usefulness in the ordinary occupations of

¹See an Editorial on "Incentives to mathematical activity," *AMERICAN MATHEMATICAL MONTHLY*, 1913, 169-173.

life, no general brief for algebra and geometry can be defended; that on the score of training special faculties, such as memory and reasoning power, the acquirements in mathematics do not carry over in general to other fields; that on the score of general culture the claims of mathematics which have held it as a required course in the curriculum have been grossly exaggerated. Hence, they conclude that, except for those looking forward to technical courses or to teaching mathematics, algebra and geometry should be removed from the curriculum as required subjects.

It is not my purpose to discuss here the validity of any of these claims against mathematics—there has been much discussion pro and con in the public press. Suffice it to say that, while in some school systems action has been taken toward eliminating or greatly reducing the required work in mathematics, nevertheless, the public in general is not convinced of the wisdom of such action, the psychologists are not agreed among themselves as to their claims, and the conclusions drawn from systems of mental tests are far from convincing. It is doubtless true that the personal experiences of a large multitude of individuals belie the charges against mathematics and hence the thinking public stands unconvinced.

However, no one would claim that the content and method of presentation of our mathematical courses, whether in elementary school, high school, or college, are beyond improvement; in fact, probably the most merciless critics of content and method are the mathematicians themselves. In no other branch of the curriculum has there been so much thoughtful agitation for improvement. It is well that the plans for improvement are in the hands of experts in the mathematical field. This is the service which the Mathematical Association of America has rendered to the cause in establishing the National Committee on Mathematical Requirements. We have faith to believe that the present uncertain state in the elementary and secondary fields will be satisfactorily adjusted through the action and influence of this National Committee.

Meanwhile, the Association, through the MONTHLY, through its national meetings and the meetings of its ten sections, through the widening circle of undergraduate mathematical clubs, through the expanding influence of its members as they become more and more conscious of the opportunity for improvement in the collegiate field and more and more enthused with zeal for promoting the interests of mathematics both pedagogically and scientifically, will go forward vicariously fulfilling the prophetic vision which Professor Moore had in mind for the Society in 1902 when he said, in his presidential address: "Do you not feel with me that the American Mathematical Society, as the organic representative of the highest interests of mathematics in this country, should be directly related with the movement for reform? And, to this end, that the Society enlarging its membership by the introduction of a large body of the strongest teachers of mathematics in the secondary schools, should give continuous attention to the question of improvement of education in mathematics, in institutions of all grades?" Thus, what Professor Moore and many others had hoped would be the rôle of the Society, has, by mutual and satisfactory arrangements, fallen to the lot of the Association.

In conclusion, let me speak briefly of the serious economic conditions which confront the Association in common with the Society and most other scientific organizations. The great increase in the cost of printing and supplies will doubtless make necessary an advance in the annual dues of the Association by at least thirty-three and one third per cent. [an advance since authorized]. This seems a modest increase in comparison with most other kinds of expenses, but it will be a real burden for a large number of members in the colleges and universities where salary advances have not kept pace with the increased cost of living. Moreover, this increase will scarcely meet the added expense for printing without cutting down the number of pages in the MONTHLY and this at a time when the amount of high-grade material on hand for publication is much greater than ever before, and when plans were already developing to increase the number of issues to eleven and eventually to twelve by adding one or two numbers devoted mainly to expository and historical articles of an elementary character.

But even all this is by no means the most serious phase of the situation. It is clear that the time has come when we should no longer allow one or two members of the Association, merely as a labor of love, to perform the very arduous services which devolve upon them. This was doubtless unavoidable in the early years of our development before the full significance of the movement which the MONTHLY inaugurated and the Association perpetuated, was widely comprehended. But the sacrifices and whole-hearted devotion given to a cause by its pioneers should not be presumed upon as indicating the normal method of procedure when the cause has become well established and its supporters are large in number.

To be more explicit, the responsibilities carried by our Secretary-Treasurer and by our Editor-in-Chief are burdensome beyond the belief of anyone who has not had actual experience in such matters, and we have no justification as a body in allowing such time and strength-consuming service to be given without compensation. We would all declare with enthusiasm that such service is invaluable and that its influence is of far-reaching importance, but we should also realize that the time given to it—amazing in amount, if we only knew all the facts—is taken at the expense of necessary recreation or of professional growth and advancement or of both. All of us expect to render a reasonable amount of altruistic service, but few of us are in a position to afford to devote a very large fraction of our time to such service for more than a brief period during some stress of circumstances, such, for instance, as happened during the war. In a word, I am advocating that the secretaryship and the chief editorship in such organizations as the Association and the Society should be offices with at least a moderate stipend attached. This, of course, is impossible at present in the Association, and always will be impossible so long as the dues are kept within the present range of reasonable reach of the average member.

One thing, however, the Council has decided must be done, namely, adequate paid assistance must be provided for these offices and the funds therefor must be found. Some paid assistance has, of course, been provided, but quite inadequate, either for the Secretary's office or for the editorial office. The Council

has voted to pay \$100 per month, beginning at once, for expert editorial and clerical assistance in Professor Archibald's office. This will cancel all our reserve before 1922, unless we secure other income than that derived from the dues. This I believe can be done. A temporary subsidy fund can be raised by the voluntary contributions of members or of other friends of mathematics. When it becomes known that a definite service may be rendered to the cause of mathematics by such contributions, interested persons will come forward and offer assistance. Such contributions will tide over a present difficult situation, but what is needed is a permanent endowment fund. This also can be secured, but it will take more time. The reason that such endowments have not been made before is that the need and the opportunity have not been known. Steps have already been taken to incorporate the Association so that we may be legally authorized to handle such trust funds.¹ An endowment can be built up from many sources as the years advance. The Association will thus become able to pay for important service rendered without advancing dues beyond the reach of the average college teacher, and it will be able to magnify its work and influence in a multitude of ways, some of which have been described above, but more of which will develop as those in hand are worked out. Opportunities will be found for service not only in the current and routine affairs of the Association, but also in the greater undertakings such as the proposed Dictionary of Mathematical Terms, which is already definitely projected and awaiting funds, or a biographical dictionary or a history of mathematics, which may later become definite projects, any one of which would constitute an enduring monument to the person who should provide the funds to make such a project attainable. It seems not too much to hope that within a reasonable time the Association, perhaps in combination with the Society, should have a central office where its business and editorial affairs of whatever magnitude may be conducted efficiently and with just compensation to those who render such important service.

I have thus endeavored to describe some of the mathematical activities in America during the past twenty years and to show the relation of the Association to these activities during its brief history, first in regard to research scholarship in its broader sense and secondly, in regard to the teaching of mathematics in all its phases. I have tried to show that the Association has put into effective operation a really national investigation of the mathematical situation in the schools, conducted by experts in both teaching and scholarship whose findings and recommendations must inspire confidence and promote reforms wherever they may be needed. And, finally, I have tried to emphasize the pressing need of financial support for such an organization as the Association through subsidies and endowments which shall make possible the conduct of its great altruistic service without, on the one hand, unjustly overburdening the volunteer workers, and without, on the other hand, raising the annual dues beyond the reasonable reach of large numbers to whom the service of the Association is especially helpful. It is the privilege and duty of every member of the Association to make known this need and opportunity as widely as circumstances permit.

¹ The legal steps have since been completed.